**A person in a white shirt

Description automatically generatedA person in a suit holding a cigarette

Description automatically generatedA close-up of a person smiling

Description automatically generatedA person holding an object

Description automatically generatedA person and person sitting in a room

Description automatically generated**

**Case Study: Movie Production Optimization**

**Context:**

A movie production studio is planning to release two blockbuster movies, M1 and M2​. Producing these movies requires three critical resources:

1. **Directors**: A pool of skilled directors who oversee movie production.
2. **Actors**: A group of versatile actors who perform in the movies.
3. **Studios**: State-of-the-art studio spaces to shoot the films.

Each movie generates a profit:

* M1​: $70,000,
* M2​: $100,000.

The available resources and resource consumption for each movie are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Resource | Available Hours | M1​ Usage | M2​ Usage |
| Directors | 300 hours | 6 hours | 8 hours |
| Actors | 500 hours | 10 hours | 15 hours |
| Studios | 200 hours | 4 hours | 5 hours |

**Objective:**

The studio wants to **maximize profit** while ensuring it does not exceed the available hours of directors, actors, and studios.

**Questions for Students**

1. **Solve the Primal Problem**:
   * Use a software tool like Python (with cvxpy) or Excel Solver to determine:
     + The **optimal profit** (Z),
     + The number of Movie 1 (M1) and Movie 2 (M2​) units to produce.
2. **Solve the Dual Problem**:
   * Calculate the shadow prices (Y1,Y2,Y3) and the **optimal resource cost** (W).
3. **Validate Strong Duality**:
   * Verify that Z=W, ensuring strong duality holds.
4. **Interpret the Shadow Prices**:
   * Explain the economic meaning of Y1,Y2,Y3​. For example:
     + How much would the profit increase if the studio had one more hour of directors?
5. **Resource Utilization**:
   * Check which resources are fully utilized (binding constraints) and which are not fully used (non-binding constraints).

**Primal Problem Formulation**

1. **Decision Variables**:
   * M1​: Number of Movie 1 units to produce.
   * M2​: Number of Movie 2 units to produce.
2. **Objective Function**:

Maximize Z=70000M1+100000M2

1. **Constraints**:
   * Directors: 6M1+8M2≤300,
   * Actors: 10M1+15M2≤500,
   * Studios: 4M1+5M2≤200,
   * Non-negativity: M1,M2≥0.

**Dual Problem Formulation**

1. **Dual Variables**:
   * Y1: Shadow price (cost) for Directors,
   * Y2​: Shadow price (cost) for Actors,
   * Y3​: Shadow price (cost) for Studios.
2. **Objective Function**:

Minimize W=300Y1+500Y2+200Y3

1. **Constraints**:
   * Coefficient of M1​: 6Y1+10Y2+4Y3≥70000
   * Coefficient of M2​: 8Y1+15Y2+5Y3≥100000,
   * Non-negativity: Y1,Y2,Y3≥0.